

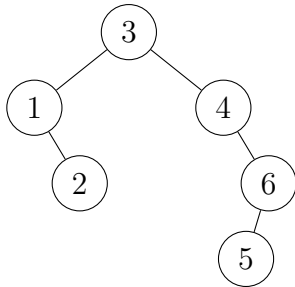
Name: \_\_\_\_\_

1. (1 pt.)

- **Read all material carefully.**
- You may refer to your books, papers, and notes during this test.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use the conventions used in class and the textbook for notation, algorithmic options, etc.

Write your name in the space provided above.

2. (9 pts.) Provide a list of keys that produce the following binary search tree when they are inserted into an initially empty tree in list order. Depict the state of the tree after each insertion.



3. (10 pts.) Provide *all* lists that satisfy the requirement of Question 2. Justify your answer.

4. (10 pts.) For each value of  $n = 1, 2, 3, 4$ , depict *all possible binary search trees* containing the  $n$  keys  $1, 2, \dots, n$ . *Justify your answer* briefly, explaining why the trees you depict are the only ones possible.

5. (5 pts.) We represent the empty binary tree by  $\perp$  and a nonempty binary tree with root  $n$ , left subtree  $l$ , and right subtree  $r$  by the triple  $(n, l, r)$ . Consider the following function  $f$  on binary trees:

$$f(t) = \begin{cases} (n, \perp, \perp) & \text{if } t = (n, \perp, \perp) \\ (n, \perp, f(l)) & \text{if } t = (n, l, \perp) \text{ and } l \neq \perp \\ (n, f(r), \perp) & \text{if } t = (n, \perp, r) \text{ and } r \neq \perp \\ (n, f(l), f(r)) & \text{if } t = (n, l, r) \text{ and } l, r \neq \perp \\ \perp & \text{otherwise} \end{cases}$$

Depict, using the usual graphical conventions, the binary tree  $f(T)$  where  $T$  is the tree of Question 2.

6. (5 pts.) We use the notation  $f^k(t)$  (with  $k > 0$ ) to denote  $k$  nested applications of the function  $f$ , that is,  $f(f(f(\dots f(t))))$ , where there are  $k$  instances of  $f$  in the expression. Using the definitions of  $f$  and  $T$  from Question 5, depict, using the usual graphical conventions, the binary trees  $f^{20}(T)$  and  $f^{21}(T)$ . *Explain your answers.* (There is no credit for answers without proper explanations.)