Name: _____

1. (1 pt.)
   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use the conventions used in class and the textbook for notation, algorithmic options, etc.

   Write your name in the space provided above.

2. (9 pts.) Depict the sequence of AVL-tree states resulting from the insertion of the following keys, in the order presented, into an initially empty tree. You must depict the state of the tree after each insertion, clearly marking and identifying (single, double) any rotations used.

$$50, 30, 40, 60, 70, 38, 36, 41, 42, 43$$

[additional space for answering the earlier question]

3. (10 pts.) Repeat Question 2 for *red-black* trees. *Follow the graphical conventions used in class:* round nodes for red and boxed nodes for black.

4. (5 pts.) What is the number of (nonisomorphic) *perfect binary search trees* on keys $1, 2, \ldots, n$ with $n < 100$? Justify your answer.

5. (5 pts.) We call a binary search tree *RB-able* if there exists an assignment of colors to its vertices that yields a red-black tree. Provide, with brief explanation, an example of a binary search tree that is an AVL tree but not RB-able, or explain why no such tree exists.