Name: ____

1. (1 pt.)

   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use textbook and classroom conventions for notation, algorithmic options, etc.
   - Ask for clarifications on the above if needed.

   Write your name in the space provided above.

2. 4 pts. Using terminology from Reynolds's paper,[1] list the first 12 k-generalized Fibonacci numbers for $k = 5$.

3. (10 pts.) Depict (using a tabular format as done in class) the action of a 5-way (6-tape) *polyphase* merge with the following initial run distribution. Each row of the table should indicate the number of runs on each tape after the completion of a phase.
   10 8 6 4 4 0

[1]Samuel W. Reynolds, "A Generalized Polyphase Merge Algorithm," *Communications of the ACM* 4/8 (1961).

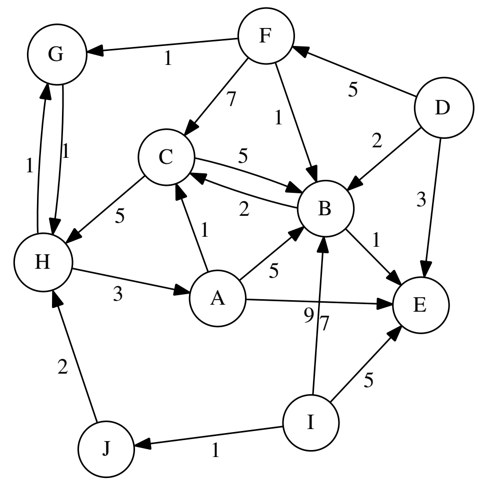
4. (15 pts.) Consider the successive insertion of each of the following keys into an initially empty *binary heap* (min-heap):

   49 98 50 84 45 69 81 36 51 61

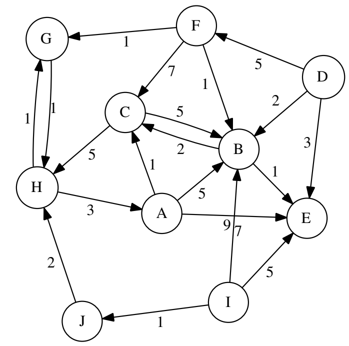
   Depict the state of the heap after each insertion in both implicit, array form and explicit, graphical tree form.

5. (10 pts.) Depict the heap resulting from a *deleteMin* operation on the final heap of Question 4. Repeat for three more deleteMin operations.

6. (25 pts.) Trace the execution of Dijkstra's shortest-path algorithm, with a pairing heap, from source vertex A on the following graph. Follow the textbook's algorithm and conventions. Depict (1) the D_w values and (2) the state of the heap after the computation of the shortest path to each vertex. Note that edge AE has weight 9 and and IB has weight 7.



[additional space for answering the earlier question]



7. (20 pts.) Depict the result of inserting the following keys, in the order presented, into an initially empty *B-tree* with parameters $M = 3$ and $L = 3$, based on the definitions and methods in the textbook.[2] (The tree is thus a B^+-tree.)

$$50, 90, 30, 60, 61, 62, 63, 31, 33, 32, 29, 88, 82, 79$$

Depict some intermediate states of the tree, *including at least the states after each node-splitting operation.*

Similarly, depict the result of deleting the following keys, in this order, *depicting at least the intermediate states after each node-merging operation.*

$$29, 30, 31, 32$$

[2] Mark Allen Weiss, *Data Structures and Problem Solving Using Java*, 4th edition (Addison-Wesley, 2010), §19.8.

[additional space for answering the earlier question]

8. (15 pts.) A warehouse for a online shopping site uses a robot to process incoming prioritized requests for items. The priority of each request is a positive integer, with higher numbers denoting higher priority. There may be multiple requests with the same priority. The robot can process only a single request at a time and it is required to process requests in priority order: The robot must not process a request of priority p if there is an unprocessed request of priority $q > p$. If two requests have the same priority then they must be processed in the order they were received.

   Describe efficient algorithms and data structures for a program that accepts two kinds of requests: (a) `addRequest(`r, p`)`, which adds a request r with priority p to the robot's collection of requests to process. (b) `nextRequest()`, which returns next request to be processed given the above constraints, and removes that request from the collection of pending ones.

   Try to choose the most efficient options from those studied in this course. Justify your choices briefly.

[additional space for answering the earlier question]