Name: _____

1. (1 pt.)
   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use the conventions used in class and the textbook for all material.

   Write your name in the space provided above.

2. (9 pts.) A *proper k-coloring* of a graph is an assignment of *colors* $1, 2, \ldots, k$ to the vertices of the graph such that no two neighboring vertices have the same color. A graph is said to be *k-colorable* if it admits a proper k-coloring.

   Write a Datalog query that tests the 2-colorability of a connected graph that is represented by its edges in a relation `Edges(s,d)`. Briefly explain why your query is correct.

   Hint: A graph is 2-colorable iff it does not contain a cycle of odd length.

3. (10 pts.) Provide recursive-SQL query that is equivalent to the query of Question 2. Briefly explain why your query is correct.

4. (20 pts.) Consider a relation $R(A, B, C, D, E, F)$ with the following basis of dependencies (note carefully: FDs v. MVDs):

$$\begin{aligned} A &\rightarrow BC \\ CD &\rightarrow A \\ D &\rightarrow E \\ F &\rightarrow B \\ AC &\twoheadrightarrow E \end{aligned}$$

   (a) Provide an instance of R that *violates* the dependency $CD\rightarrow A$ without violating any of the other dependencies.

   (b) List **all** keys of R.

   (c) Explain your answer, noting why the keys you list are valid and also why there are no other keys.

   (d) How many *superkeys* does R have? Explain your answer. (You need not list all superkeys.)

5. (15 pts.) Decompose the schema of Question 4 to 4NF. Show all intermediate steps and details, as in class exercises (keys, projected dependencies, decomposed relations, etc.).

$$\begin{aligned} A &\rightarrow BC \\ CD &\rightarrow A \\ D &\rightarrow E \\ F &\rightarrow B \\ AC &\twoheadrightarrow E \end{aligned}$$

[additional space for answering the earlier question]

6. (5 pts.) Given a database with table `R(A, B, C, D, E, F)`, with all attributes of type integer, provide the simplest SQL statements to declare the following constraints:

   (a) Attributes `A` and `B` must not be null.

   (b) The sum of `C` and `D` must be no greater than `E`.