

Name: _____

1. (1 pt.)

- **Read all material carefully.**
- You may refer to your books, papers, and notes during this test.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use textbook and classroom conventions for notation, algorithmic options, etc.
- Ask for clarifications on the above if needed.

Write your name in the space provided above.

2. (14 pts.) Depict **all** red-black trees that contain exactly the five keys 1, 2, 3, 4, and 5. As in class, use circles for red nodes and boxes for black nodes. Briefly **explain** why the trees you depict are the only possibilities. If there are more than 10 trees that qualify, depict any 10 of your choice.

[additional space for answering the earlier question]

3. (15 pts.) For each tree of Question 2:

- (a) Redraw the tree below ignoring node colors. Depict each node as a circle enclosing its key.
- (b) Write the height of each node to the right of its circle.
- (c) If the resulting tree is a valid AVL-tree, write *AVL* next to its root. Otherwise, write *not AVL* next to its root and mark AVL-unbalanced nodes with an asterisk ***.

4. (15 pts.) [This question is similar to Question 3, but uses a mapping to AA trees instead of to AVL trees.] For each tree of Question 2:
- (a) Redraw the tree below with each red node drawn as a horizontal child of its parent (cf. AA-trees); then ignore colors. Black nodes remain vertical children of their parents as before. Depict each node as a circle enclosing its key and be sure to depict horizontal children clearly.
 - (b) **If** the resulting tree is a valid AA-tree **then**:
 - i. Write *AA* next to its root.
 - ii. Write the AA-tree level of each node to the right of its circle.**else**:
 - i. Write *not AA* next to its root.
 - ii. Mark with an asterisk * all feature that violate AA-tree properties.

[additional space for answering the earlier question]

5. (15 pts.) For each *valid* AA-tree of Question 4:
- (a) Present a list of keys that produces the tree when inserted in sequence into an initially empty tree.
 - (b) Depict the action of the above insertions. Mark any skew and split operations used. Depicting the state of the tree after each insertion.

[additional space for answering the earlier question]