A THEORY OF PROGRAM SIZE FORMALLY IDENTICAL TO INFORMATION THEORY

Abstracts of Papers, 1974 IEEE International Symposium on Information Theory, October 28–31, 1974, University of Notre Dame, Notre Dame, Indiana, USA, IEEE Catalog No. 74 CHO 883–9 IT, p. 2

Monday, October 28, 11:00 AM—Plenary Session A (continued)

A Theory of Program Size Formally Identical to Information Theory, Gregory J. Chaitin (IBM World Trade Corporation, Buenos Aires, Argentina).

A new definition of the program-size complexity is made.

$$H(A, B/C, D)$$

is defined to be the size in bits of the shortest self-delimiting program for calculating the strings A and B if one is given a minimal-size self-delimiting program for calculating the strings C and D. This

differs from previous definitions: (1) programs are required to be self-delimiting, i.e., no program is a prefix of another, and (2) instead of being given C and D directly, one is given a program for calculating them that is minimal in size. Unlike previous definitions, this one has precisely the formal properties of the entropy concept of information theory. For example,

$$H(A/B) = H(A,B) - H(B) + O(1).$$

Also, if a program of length k is assigned measure 2^{-k}, then

$$H(A) = -\log_2 \left(\begin{array}{c} \text{the probability that the standard} \\ \text{universal computer will calculate } A \end{array} \right) + O(1).$$