

Name: _____

1. (1 pt.)

- **Read all material carefully.**
- *If in doubt whether something is allowed, ask, don't assume.*
- You may refer to your books, papers, and notes during this test.
- E-books may be used *subject to the restrictions* noted in class.
- Computers are not permitted, except when used strictly as ebooks.
- Network access of any kind (cell, voice, text, data, ...) is not permitted.
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use class and textbook conventions for notation, algorithmic options, etc.

Write your name in the space provided above.

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| WAIT UNTIL INSTRUCTED TO CONTINUE TO REMAINING QUESTIONS. |
|---|

Do not write in the following table.

| Q | Full Score |
|-------|------------|
| 1 | 1 |
| 2 | 9 |
| 3 | 20 |
| 4 | 10 |
| total | 40 |

2. (9 pts.) Draw the subproblem graph for MEMOIZED-CUT-ROD(p, n) with $n = 10$ and the following array p .

| | | | | | | | | | | | |
|---------------|---|---|---|---|---|---|----|----|----|----|----|
| length i : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| price p_i : | 0 | 2 | 3 | 7 | 8 | 9 | 14 | 15 | 16 | 17 | 20 |

3. (10 pts.) Recall the following algorithm for the cut-rod problem that was briefly discussed in class: Compute the value of price per unit length ($v(i)$) for each integer value of length $i \leq n$ (where n is the length of the rod given as input). Then repeatedly cut the remaining portion m of the rod at length $i \leq m$ with maximum $v(i)$.

Does this algorithm always produce the correct (optimal) solution?

If so, provide at least an informal proof or detailed explanation.

If not, provide and explain a small counterexample.

4. (20 pts.) Trace the execution of the MEMOIZED-CUT-ROD(p, n) algorithm for $n = 10$ and the following pricing array of Question 2, repeated below.

Depict the state of the array r at least at three points during the execution:

- (a) The first time MEMOIZED-CUT-ROD-AUX(p, n, r) returns when invoked with $n = 3$.
- (b) As above, but with $n = 7$.
- (c) At the very end.

| | | | | | | | | | | | |
|---------------|---|---|---|---|---|---|----|----|----|----|----|
| length i : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| price p_i : | 0 | 2 | 3 | 7 | 8 | 9 | 14 | 15 | 16 | 17 | 20 |

[additional space for answering the earlier question]

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