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1. (1 pt.)

- **Read all material carefully.**
- *If in doubt whether something is allowed, ask, don't assume.*
- You may refer to your books, papers, and notes during this test.
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use class and textbook conventions for notation, algorithmic options, etc.
- For the duration of the exam, the only communication (live or network) should be with the instructor for clarifications, etc.
- At the end of the exam, scan your work to a PDF file named using the following template and upload it in the usual way:

cos454-mt02-*lastname*-*firstname*-*pqrs*.pdf

(replacing *lastname* and *firstname* with yours and *pqrs* with an arbitrary 4-digit number).

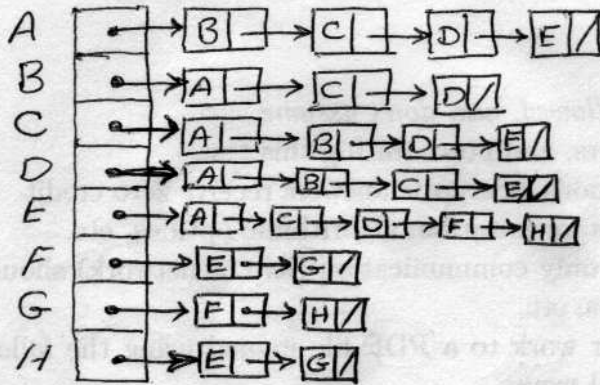
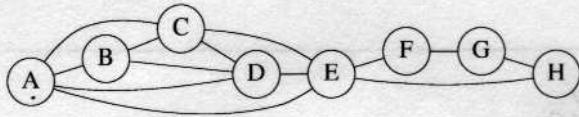
Write your name in the space provided above.

WAIT UNTIL INSTRUCTED TO CONTINUE TO REMAINING QUESTIONS.

Do not write in the following table.

| Q | Full Score |
|-------|------------|
| 1 | 1 |
| 2 | 9 |
| 3 | 10 |
| 4 | 20 |
| total | 40 |

2. (9 pts.) Depict both the *adjacency-list* and the *adjacency-matrix* representation of the graph depicted below:



| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| B | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| D | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| E | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| F | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| G | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| H | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

3. (10 pts.) Solve the following recurrence using both

(a) the master method; and

(b) either the recursion tree or the substitution method (your choice).

Show enough work to make it obvious how each method is being applied.

$$T(n) = 3T(n/5) + 17\sqrt{n}$$

[Hint: $\sqrt{5} \approx 2.236$ (so a calculator shouldn't be needed but you may use one).]

(a) $T(n) = aT(n/b) + f(n)$

$a=3, b=5,$

$f(n) = 17\sqrt{n} = 17n^{0.5}$

$n^{\log_b a} = n^{\log_5 3}$

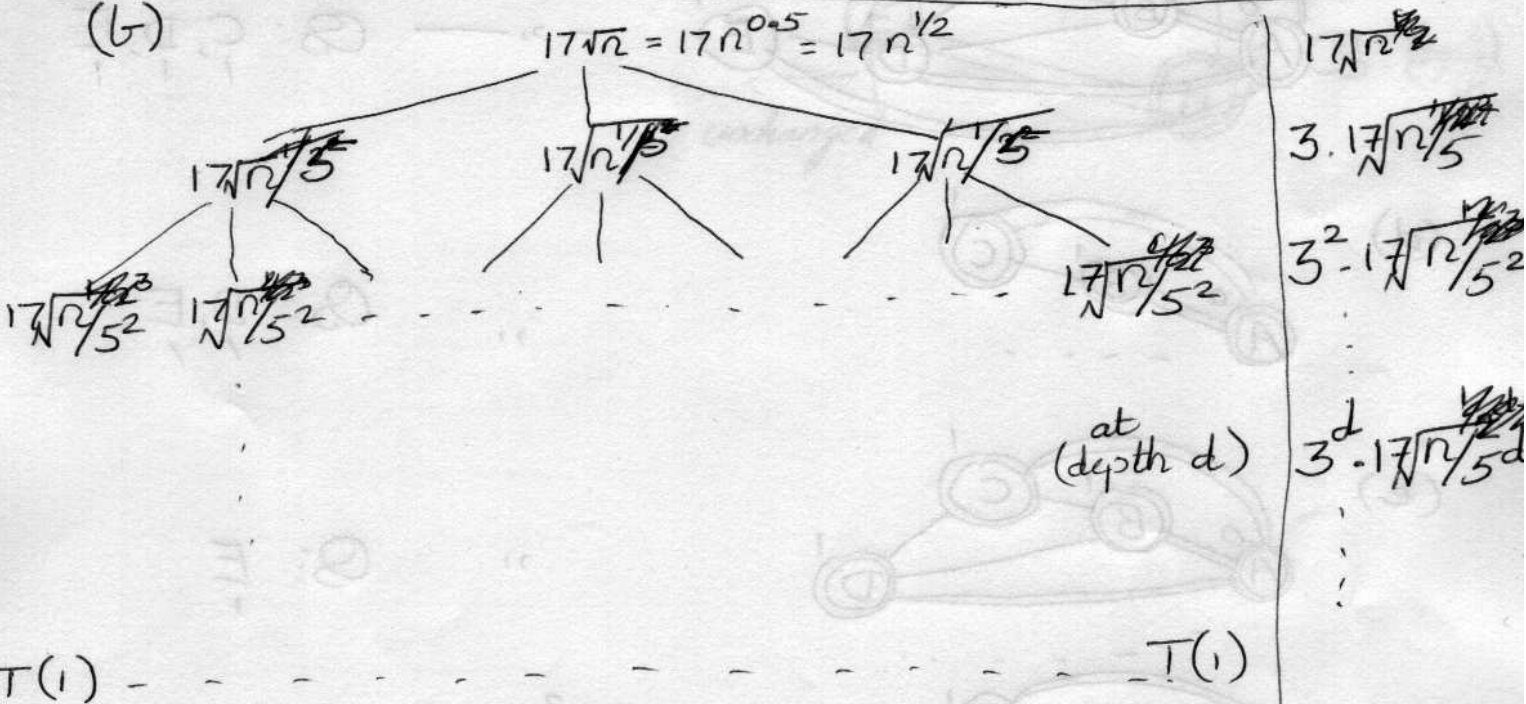
$\log_5 3 > 0.5 \Rightarrow n^{\log_5 3} > n^{0.5}$

$\Rightarrow f(n) = O(n^{\log_5 3})$

$T(n) = \Theta(n^{\log_5 3})$

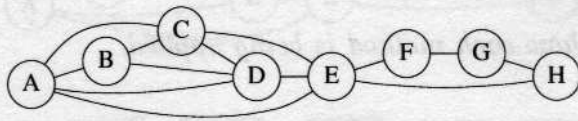
(case 1)
of thm.

(b)

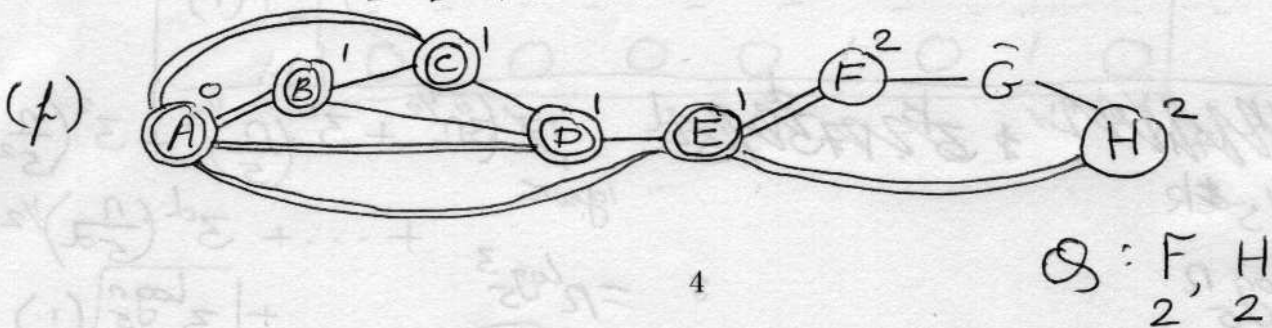
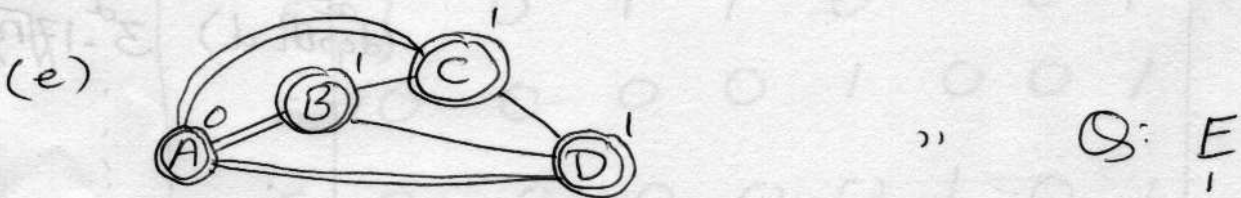
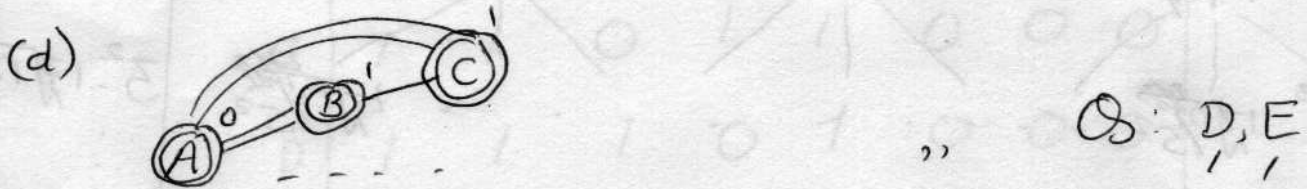
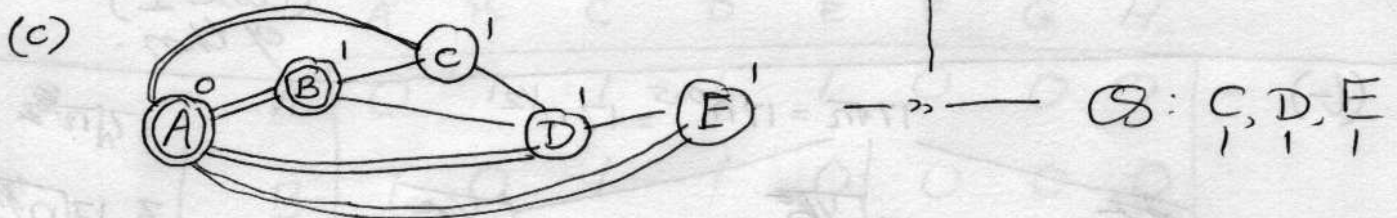
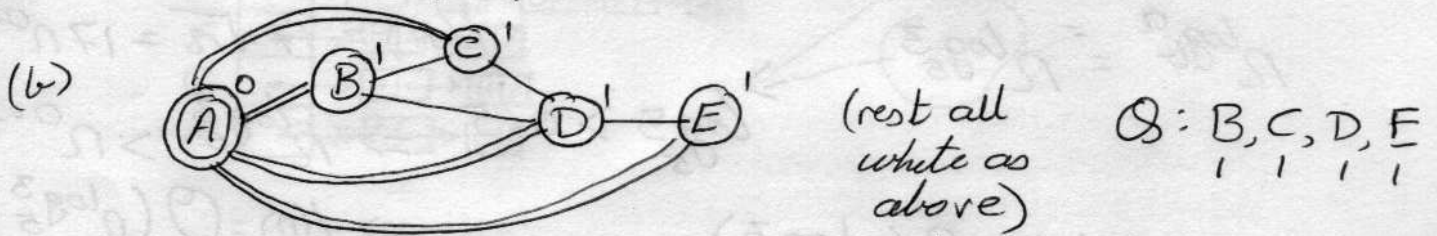
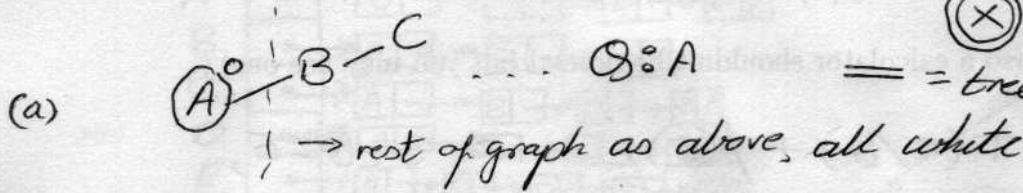


$n = \frac{n}{5^k} \Rightarrow k = \log_5 n$
 $17(3^0 n^{1/2} + 3^1 (\frac{n}{5})^{1/2} + 3^2 (\frac{n}{5^2})^{1/2} + \dots + 3^d (\frac{n}{5^d})^{1/2})$
 $= n^{\log_5 3} + \boxed{3^{\log_5 n}} (1)$

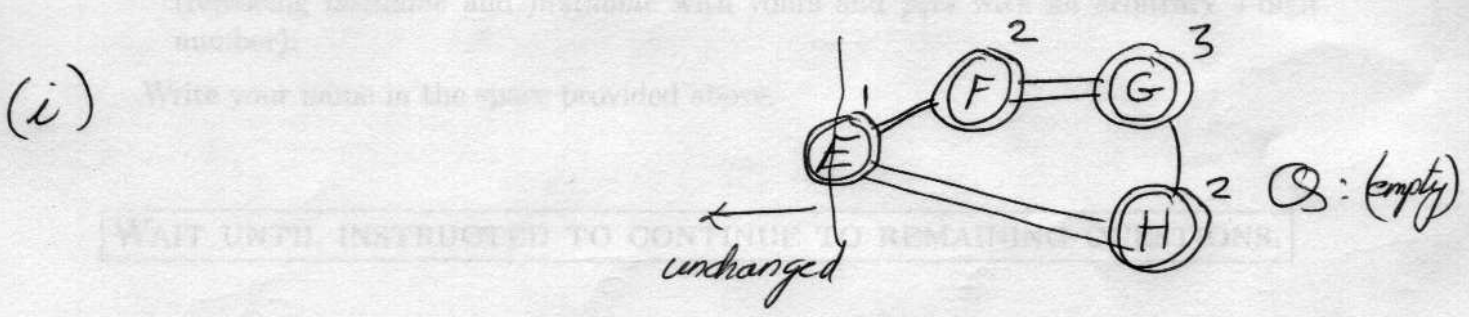
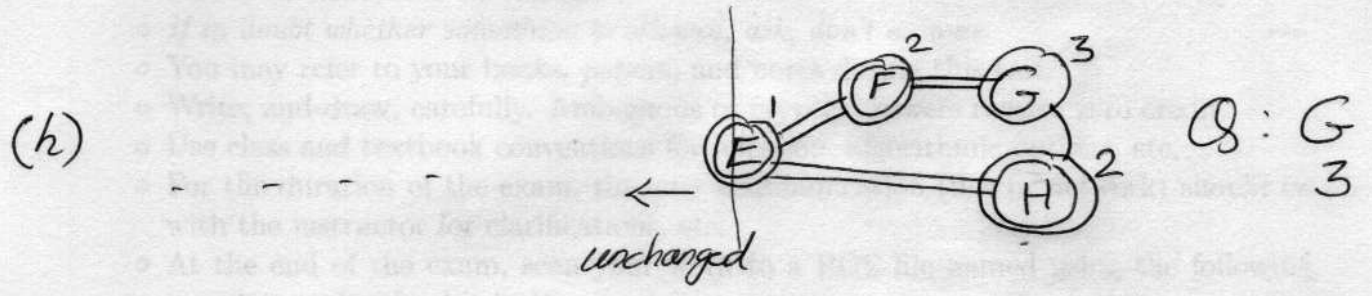
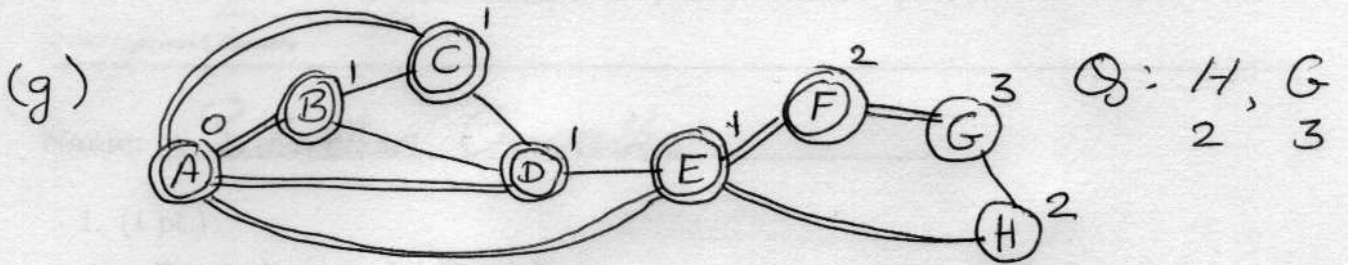
4. (20 pts.) Trace the operation of BFS, with initial vertex A, on the following graph using the conventions of Figure 22.3 in the textbook. Ensure that your work (1) indicates the color of each node in some clear manner, (2) highlights the edges of the BFS tree, and (3) depicts both the graph and the queue.



x = white
 $(x)^d$ = grey " d " distance
 $(\otimes)^d$ = black (∞ if missing)
 == tree edge



[additional space for answering the earlier question]



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