COS 454/554 Spring 2021 *Midterm Exam 1*   35 pts.; 35 minutes; 3 Qs; 6 pgs.   2021-03-08 11:00 a.m.

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Name: _____

1. (1 pt.)
   ○ **Read all material carefully.**
   ○ *If in doubt whether something is allowed, ask, don't assume.*
   ○ You may refer to your books, papers, and notes during this test.
   ○ Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   ○ Use class and textbook conventions for notation, algorithmic options, etc.
   ○ For the duration of the exam, the only communication (live or network) should be with the instructor for clarifications, etc.

   Write your name in the space provided above.

WAIT UNTIL INSTRUCTED TO CONTINUE TO REMAINING QUESTIONS.

Do not write in the following table.

Q	Full	Score
1	1	
2	14	
3	20	
total	35	

2. (14 pts.) Solve the rod-cutting problem for a rod of length 12 and the following price table. Use the EXTENDED-BOTTOM-UP-CUT-ROD algorithm. In particular:

(a) Depict the final states of the r and s arrays used by that algorithm.
(b) Indicate the distances of the optimal cuts from the left end of the original rod.

length:	1	2	3	4	5	6	7	8	9	10	11	12
price:	7	8	18	16	30	12	28	24	72	40	33	48

Solution

(a) Final arrays (indices 0–12):

i	0	1	2	3	4	5	6	7	8	9	10	11	12
$r[i]$	0	7	14	21	28	35	42	49	56	72	79	86	93
$s[i]$	–	1	1	1	1	1	1	1	1	9	1	1	1

Optimal revenue $r[12] = 93$.

Reconstruction: $s[12]=1,\ s[11]=1,\ s[10]=1,\ s[9]=9$ → pieces of length $1,1,1,9$.
Check: $7+7+7+72 = 93$.

(b) The optimal cuts (distances from the left end of the original rod) are at positions **1, 2, and 3**, yielding pieces of lengths 1, 1, 1, and 9.

[additional space for answering the earlier question]

3. (20 pts.)

    (a) Provide pseudocode for *linear search*. The input is an array $A[1, 2, \ldots, n]$ of integers and another integer, v, which is the searched value. The output is *nil* if there is no array element equal to v; otherwise, it is the smallest index i such that $A[i] = v$.

    (b) Prove the correctness of your pseudocode using appropriate loop invariants and other claims.

    (c) Analyze the running time of your pseudocode by following the textbook's method (Section 2.2).

[additional space for answering the earlier question]

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