Name: _____

This assignment continues the thread of *JJ's Jolly Jumping Journey*, or *J5*, from previous ones. The primary **goal** is to gain experience on mapping concrete problems to abstract ones, finding solutions to the abstract problems (using known solutions from books, papers, and other sources), and implementing the solutions in a concrete context. Secondary goals are gaining more experience with programming, documenting algorithms and programs, studying performance, and conducting and summarizing experiments.

The main problem JJ has reached the *circular chamber of columns* and must use a specific arrangement of three jewels and a lamp to summon help from the *jacketed justice* by shining the *JJ sign* into the sky. This circular chamber has a number of columns. Each column has a fixed height but is mounted on a circular track, centered on the chamber's center, that allows it to be moved to any position on that track. No two columns share a track and the tracks are separated well enough to avoid any interference when moving columns. The lamp and the three jewels, ruby, garnet, and beryl, must be placed on separate columns so that they form a straight line, in the listed order, that points above the horizon in the direction away from the lamp. The *main task* is figuring out which columns to use for the lamp and the ruby, garnet, and beryl jewels in order to produce the required straight-line arrangement. The input is the collection of pairs, with each pair denoting the radius of a column's circular track and that column's height. The desired output is a list of all 4-tuples of columns that satisfy the requirements.

Questions

1. (1 pt.) Write your name in the space provided above.

2. (9 pts.) Provide an abstract formulation of the main problem, using familiar mathematical concepts that are independent of JJ's journey or any other specific application. Describe the abstract formulation as precisely and as concisely as possible. Indicate how a solution to the abstract problem may be used to solve JJ's specific problem above.

[additional space for answering the earlier question]

3. (10 pts.) Describe an efficient algorithm for solving the problem of Question 2. Describe the algorithm in English as precisely as possible. Clearly indicate how the algorithm uses widely known solutions to the problem of Question 2, its subproblems, or related problems. Provide suitable citations for such work.

4. (10 pts.) Explain why the algorithm of Question 3 is correct.

5. (10 pts.) Provide pseudocode, using the textbook's style as a guide, for the algorithm of Question 3. Include explanatory comments and outline a proof of its correctness.

6. (10 pts.) State and justify the running time of the algorithm of Question 5 as a function of the number n of columns.

[additional space for answering the earlier question]

IO format Your program should read from *standard input* and write to *standard output*. The **input** is a sequence of $2m$ whitespace-delimited integers, to be interpreted as m disjoint pairs of adjacent integers: $(r_1, h_1), (r_2, h_2), \ldots, (r_m, h_m)$. Each pair (r_i, h_i) denotes a column of height h_i on a track of radius r_i. Your program's **output** should consist of one or more lines. The first line consists of just one integer r, which is the number of arrangements of columns that satisfy the requirements noted earlier. This line is followed by r lines, where each line lists the track-radii of the columns used for the lamp and the ruby, garnet, and beryl crystals (in that order) separated by single spaces. These r lines should appear in lexicographically sorted order.

Example If the input is

```
4 7      1
1 5   9   2
    3
```

then the desired output is

```
1
1 2 4 5
```

Use the newsgroup for more examples.

7. (150 pts.) Implement the algorithm. Test and document your work carefully and submit your packaged source code and supporting documentation.

8. (20 pts.) Conduct a brief experimental study of your implementation, measuring the running time for a suitable collection of inputs. Include your test code in your electronic submission, with suitable documentation.

9. (30 pts.) Summarize your experimental results by making effective use of charts and tables. Comment on how well the experimental results match the predictions based on your answer to Question 6. Highlight any significant differences and explain them the best you can. Include these results, comments, and explanations as a single PDF file in your submission.