Today: binary heaps, heapsort, external sorting; §§21.4–21.end. Reynolds's paper.[1]
Next class: Splay trees; §§22.0–22.4.
Reminders: Portfolios and posters soon.

1. List the members of your group below. Underline your name.

2. Depict the action of *in-place heapsort* on the following array. Depict the *state of the array*, and the *implicit binary heap it encodes* (in the usual graphical form), after each *deleteMin* operation.

   `66 70 99 43 58 77 41 74 81 70`

[1] Samuel W. Reynolds, "A Generalized Polyphase Merge Algorithm," *Communications of the ACM* 4/8 (1961).

3. Consider an external storage device with a *page size* of 16 bytes (artificially low for illustration). Depict an aligned layout of the following array of 32-bit integers in such a device. How many page accesses are needed to access (1) all elements and (2) every fourth element of the array?

   `83 10 80 85 88 7 93 25 63 46 9 14 87 97 49 82`

4. Depict the action of *heap sort* on the array of Question 3. Determine the exact number of page accesses made in the first three iteration of the outer loop. What is the total number of of page accesses?

   `83 10 80 85 88 7 93 25 63 46 9 14 87 97 49 82`

[additional space for answering the earlier question]

5. Depict the action of 2-way merge sort on the data of Question 4. Determine the exact number of page accesses for the first three merges, as well as the total number of page accesses.

   `83 10 80 85 88 7 93 25 63 46 9 14 87 97 49 82`

6. Repeat Question 5 for 4-way merge sort.

7. $\star$ How many page access are made by a k-way merge sort operating on nm items in external memory of page size m (as a function of k, m, and n)?

8. Fill in the blank entries in the following tables, indicating the number of runs on each of the five tapes used in a *polyphase merge-sort of order 4*. Row n of each table summarizes the distribution of runs on the tapes immediately following the nth merge, with the 0th row summarizing the initial distribution of runs (before any merges). Leave space in each cell to answer Question 9.

	# runs on tape				
m	1	2	3	4	5
0	8 (1)	8 (1)	7 (1)	7 (1)	0
1					
2					
3					
4					
5					
6					
7					
8					

	# runs on tape				
m	1	2	3	4	5
0	10 (1)	9 (1)	5 (1)	6 (1)	0
1					
2					
3					
4					
5					
6					

9. Augment the entries in the tables of Question 8 by adding (parenthesized) the sizes of the sorted runs in each cell, assuming all initial runs have 1000 records.

10. Using the definition in Reynolds's paper, list the first 20 k-generalized Fibonacci numbers for $k = 2, 3, 4, 5$.

11. Using the method suggested by Reynolds's paper, determine the initial distribution of 82 runs on 6 tapes for a 5-way polyphase merge. That is, indicate the number of runs initially written to each of the tapes, numbered 1 through 6. Show the intermediate steps used in arriving at the final distribution. Then indicate the result of each merge step in tabular form, as in Question 8, until only one run remains.