

Name: _____

1. (1 pt.)

- **Read all material carefully.**
- You may refer to your books, papers, and notes during this test.
- E-books may be used subject to the restrictions noted in class.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use class and textbook conventions for notation, algorithmic options, etc.
- There is an one extra-credit question (marked with ★). It is harder than the rest.
- Write your name in the space provided above.

2. (9 pts.) Depict the AVL tree resulting from the sequential insertion, in the listed order, of

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

into an empty tree. Depict the state of the tree before and after each insertion, as well as before and after each rotation. Indicate which rotations are used and where by annotating the affected trees.

[additional space for answering the earlier question]

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

[additional space for answering the earlier question]

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

3. (10 pts.) Repeat Question 2 for a bottom-up red-black tree.

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

[additional space for answering the earlier question]

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

4. (10 pts.) Repeat Question 2 for an AA tree.

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

[additional space for answering the earlier question]

8, 5, 6, 4, 3, 2, 1, 7, 10, 9

5. (10 pts.) Depict the AVL tree resulting from the sequential deletion, in the listed order, of

4, 5, 7

from the final tree of Question 2. Depict the state of the tree before and after each deletion, as well as before and after each rotation. Indicate which rotations are used, and where, by annotating the affected trees.

[additional space for answering the earlier question]

6. (10 pts.) What is the number of AA trees that hold the keys $\{1, 2, 3, 4, 5\}$ (only)? Depict them all. Justify your answer by explaining why the trees you depict are valid and also why there are no others.

7. (10 pts.) What is the maximum number of rotations required for inserting five keys into an initially empty AA tree?

Each use of a skew or split operation counts as one rotation. The maximum is over all possible choices and orderings of five keys.

Provide five keys whose insertion achieves that maximum. Demonstrate the claim by depicting the tree after each insertion. Explain why there can be no sequence that requires a greater number of rotations.

8. ★ (10 pts.) Depict the AA tree resulting from the sequential deletion, in the listed order, of

4, 5, 7

from the final tree of Question 4. Depict the state of the tree before and after each deletion, as well as before and after each rotation. Indicate which rotations are used and where by annotating the affected trees.