**Name:**

1. (1 pt.) Write your name in the space provided above.

2. (4 pts.) Prove or disprove: for all reals $p, q > 1$ and all functions f that map reals to reals, if f is $O(\log_p n)$ then f must also be $O(\log_q n)$.

3. (3 pts.) Depict (using the usual graphical representation) the binary search tree resulting from the insertion of the following keys, in the order presented. Depict the final tree only.

$$53, 87, 10, 32, 16, 4, 8$$

4. (2 pts.) Represent the tree of Question 3 using the *linear notation* for binary trees used in class and in the homework.

5. (3 pts.) Provide a sequence of keys that produces the following binary search tree when the keys are inserted into an initially empty tree in sequence order.
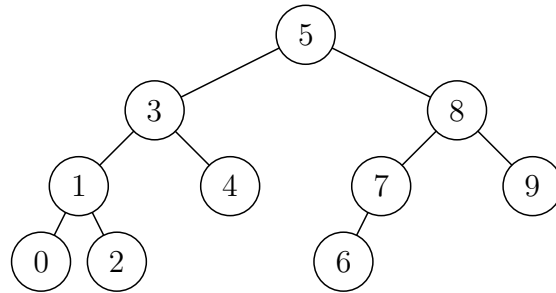
6. (5 pts.) List **all** sequences of keys that produce the following binary search tree when the keys in each sequence are inserted into an initially empty tree in sequence order. Justify your answer briefly.
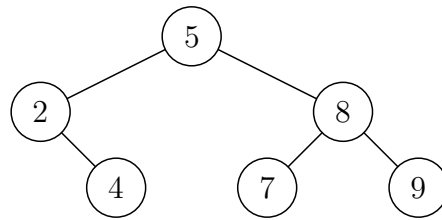
7. (2 pts.) Define *covariant type* and illustrate it using a suitable Java example.

8. (10 pts.) Depict (using the usual graphical representation) the *AVL* tree resulting from the insertion of the following keys, in the order presented. **Depict all intermediate trees.** In particular, you must depict the tree after each key is inserted, and before and after any rotations are performed. All rotations should be clearly marked and identified (single, double) and the node that is the root of the rotated subtree must also be marked (with a $*$).

$$10, 20, 30, 40, 50, 41, 42, 42, 44, 45$$

[additional space for answering the earlier question]