



5. For an integer  $n > 1$ , let  $V_n$  be the set of  $(n - 2)$ -character strings  $\{x_1x_2 \dots x_{n-2} \mid x_i \in \{1, 2, \dots, n\} \text{ and } x_i \neq x_j \text{ unless } i = j\}$ .
- (a) List  $V_n$  for  $n = 2, 3, 4$ .
  - (b) What is the cardinality of  $V_n$ , as a function of  $n$ ?
  - (c) Provide an alternate, equivalent (perhaps simpler) definition of  $V_n$ .

6. For an integer  $n > 1$ , define a digraph  $Q_n = (V_n, E_n)$  where the set of vertices  $V_n$  defined in Question 5 and the set of edges  $E_n = \{(u, v) \mid u, v \in V_n \text{ with } u = x_1x_2x_3 \cdots x_{n-2}, v = x_2x_3 \cdots x_{n-2}x_{n-1}, \text{ where } x_i \neq x_j \text{ for } i \neq j\}$ .
- (a) Depict  $Q_n$  for  $n = 2, 3, 4$ .
  - (b) What is the cardinality of  $E_n$ , as a function of  $n$ ?
  - (c) Is there anything notable about the degrees of vertices in  $Q_n$ ?
  - (d) Provide an alternate, equivalent (perhaps simpler) definition of  $Q_n$ .

7. Do the graphs  $Q_2$ ,  $Q_3$ , and  $Q_4$  of Question 6 have Eulerian paths? For each graph, exhibit an Eulerian path or explain why no such path exists.

Recall that an Eulerian path in a digraph is a directed path that traverses each edge exactly once. A digraph with such a path is called Eulerian.

8. Prove or disprove: The graphs  $Q_n$  of Question 6 are Eulerian for all  $n > 1$ .